CENG 477
Introduction to Computer Graphics
Forward Rendering Pipeline
Clipping and Culling
Rendering Pipeline

- Sequence of operations that is used to draw primitives defined in a 3D coordinate system on a 2D window
- Can be implemented on **hardware** or **software**
- Two notable APIs: OpenGL and D3D
- **Not static**: Constantly evolving to meet the demands of the industry
Rendering Pipeline – Overview

1. Get vertices in specific order and connectivity information
2. Process vertices
3. Create primitives from connected vertices
4. Clip and cull primitives to eliminate invisible ones
5. Transform primitives to screen space (preserve z)
6. Rasterize primitives to obtain fragments
7. Process fragments to obtain visible pixels with color
Until Now

- We’ve done transformations: model to world to camera to clip space
- We skip over primitive assembly as it is mostly straightforward
- We’ll look at clipping and culling today
- We’ve done viewport transformations
- Yet to come rasterization: lines, triangles, interpolation
- Yet to come fragment processing (blending, depth testing, alpha testing, etc) afterwards
Clipping

- In modern graphics API, there are essentially three kinds of primitives: **points, lines, and triangles**
- Point clipping: straightforward
  - Reject a point if its coordinates are outside the viewing volume
- Line clipping
  - Cohen-Sutherland algorithm
  - Liang-Barsky algorithm
- Polygon clipping
  - Sutherland-Hodgeman algorithm
Clipping

- Clipping is done in the **clip space** which is a result of applying projection (orthographic or perspective) transformation
- After perspective transformation the **w** component of a point becomes equal to $-z$

$$M_{per} = \begin{bmatrix} \frac{2n}{r - l} & 0 & \frac{r + l}{r - l} & 0 \\ \frac{r - l}{2n} & \frac{r - l}{t - b} & \frac{r - l}{t - b} & 0 \\ 0 & \frac{2n}{t - b} & \frac{t + b}{t - b} & 0 \\ 0 & 0 & \frac{f + n}{f - n} & \frac{f + n}{f - n} \end{bmatrix}$$
Clipping

• To find the actual point in the canonical viewing volume, we divide by this last component.

• However, clipping is performed before dividing by $w$ (that is $-z$) for two reasons:
  – $w$ may be equal to 0 in which case division would be undefined.
  – Instead of comparing $-1 \leq \frac{x}{w} \leq 1$ we can directly compare
    – $w \leq x \leq w$ thus avoiding an extra division.
  – The same goes for $y$ and $z$ components.
  – That is why in the following we don’t clip against -1 and 1 but against arbitrary numbers (it is also possible to define user clip planes which may have arbitrary values).
Clipping

- For simplicity, however, in the following we assume that clipping is performed against a 2D box with coordinates between $[x_{\text{min}}, x_{\text{max}}]$ and $[y_{\text{min}}, y_{\text{max}}]$
- The same ideas can be easily generalized to 3D
- Line clipping:
  - Cohen-Sutherland Algorithm
  - Liang-Barsky Algorithm
- Polygon clipping:
  - Sutherland-Hodgeman Algorithm
Cohen-Sutherland Algorithm

• Assign **outcodes** to the end points of lines:
  – Bit0 = 1 if region is to the left of left edge, 0 otherwise
  – Bit1 = 1 if region is to the right of right edge, 0 otherwise
  – Bit2 = 1 if region is below the bottom edge, 0 otherwise
  – Bit3 = 1 if region is above the top edge, 0 otherwise

Outcodes for this line are: 0101 and 1010

How many regions would we have in 3D?

How bits would we need?
Cohen-Sutherland Algorithm

- **Handle trivial accept and trivial rejects** first:
  - If both outcodes are zero accept the line as it is
  - If BITWISE AND of outcodes are non-zero, reject the line entirely
Cohen-Sutherland Algorithm

- For non-trivial cases, *iteratively subdivide* lines until all parts can be trivially accepted and rejected
  - Iteration follows a fixed order (e.g. left, right, bottom, top)
Cohen-Sutherland Algorithm

- Non-trivial cases:
  - **Step 1**: Pick an outside endpoint (v0 or v1)
  - **Step 2**: Determine if it is outside an edge
  - **Step 3**: Intersect it with that edge creating a new endpoint
  - **Step 4**: Assign outcode to the new endpoint and go to step 1
Cohen-Sutherland Algorithm

• Non-trivial cases:
  - **Step 1:** Pick an outside endpoint (v0 or v1)
  - **Step 2:** Determine if it is outside an edge
  - **Step 3:** Intersect it with that edge creating a new endpoint
  - **Step 4:** Assign outcode to the new endpoint and go to step 1
Cohen-Sutherland Algorithm

• May perform needless clipping as we follow a fixed order:

\[
\begin{array}{c|c|c|c}
0001 & 0000 & 0010 & v0 \\
0101 & 0100 & 0110 & v1 \\
\end{array}
\]
Cohen-Sutherland Algorithm

- May perform needless clipping as we follow a fixed order:

Trivial reject at the next step
Line Intersections

- In 2D, we need to intersect a line with other lines
- In 3D, we need to intersect a line with planes
- We may use parametric form in both cases

\[ \mathbf{v}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad x(t) = x_0 + dt \\
\mathbf{v}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad y(t) = y_0 + dt \quad \text{where} \quad \mathbf{d} = \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix} \quad z(t) = z_0 + dt \]
Line Intersections

- To find the intersection point, compute $t$ corresponding to the given edge (or face) and then find the remaining values:

\[ t_{max} = \frac{x_{max} - x_0}{x_1 - x_0} \]

The intersection point is at:

\[(x_{max}, y(t_{max}), z(t_{max}))\]
Cohen-Sutherland Algorithm

• Advantages:
  – If the chances of trivial accept/reject are high, this is a very fast algorithm
  – This can happen if the clipping rectangle is very large or very small

• Disadvantages:
  – Non-trivial lines can take several iterations to clip
  – Because testing and clipping are done in a fixed order, the algorithm will sometimes perform needless clipping
Liang-Barsky Algorithm

- Uses the idea of parametric lines
- Classifies lines as potentially entering and potentially leaving to speed up computation (approximately 40% speed-up over Cohen-Sutherland Alg.)

Goal: Given the line $v_0, v_1$ determine:
- The part of the line is inside the viewing rectangle.

Note: $p = v_0 + (v_1 - v_0)t$
Liang-Barsky Algorithm

- Potentially entering (PE) and leaving (PV):
- Why do we say potentially?

- $v_0, v_1$ is potentially entering the left edge as $x_1 - x_0 > 0$
- $v_2, v_3$ is potentially leaving the left edge as $x_3 - x_2 < 0$

The situation is reversed for the right edge:

- $v_4, v_5$ is potentially leaving the right edge as $x_5 - x_4 > 0$
- $v_6, v_7$ is potentially entering the right edge as $x_7 - x_6 < 0$
Liang-Barsky Algorithm

- Similar for bottom and top edges:

  - $v_0, v_1$ is potentially entering the bottom edge as $y_1 - y_0 > 0$
  - $v_2, v_3$ is potentially leaving the bottom edge as $y_3 - y_2 < 0$

  The situation is reversed for the top edge:

  - $v_4, v_5$ is potentially leaving the top edge as $y_5 - y_4 > 0$
  - $v_6, v_7$ is potentially entering the top edge as $y_7 - y_6 < 0$
Liang-Barsky Algorithm

- **Observation:** If a line is first leaving then entering, it cannot be visible
Liang-Barsky Algorithm

- Visible lines are first entering then leaving:
Liang-Barsky Algorithm

• Mathematical interpretation:

  \[
  \begin{align*}
  \text{if} \ (t_{PL} < t_{PE}) & : \\
  \text{visible} &= \text{false};
  \end{align*}
  \]

  Where \( t_{PL} \) is the t value for the \textbf{first} leaving intersection and \( t_{PE} \) is the t value for the \textbf{last} entering intersection.

• So at intersection points, we need to compute the t value as well as whether the line is \textbf{PE} or \textbf{PL} at that point.
Liang-Barsky Algorithm

• Computing $t$ value at every edge:

$$x_{\text{left}} = x_0 + (x_1-x_0)t \Rightarrow t = (x_{\text{left}} - x_0) / (x_1 - x_0)$$
$$x_{\text{right}} = x_0 + (x_1-x_0)t \Rightarrow t = (x_{\text{right}} - x_0) / (x_1 - x_0)$$
$$y_{\text{bottom}} = y_0 + (y_1-y_0)t \Rightarrow t = (y_{\text{bottom}} - y_0) / (y_1 - y_0)$$
$$y_{\text{top}} = y_0 + (y_1-y_0)t \Rightarrow t = (y_{\text{top}} - y_0) / (y_1 - y_0)$$

• But this does not help us to know if line is entering or leaving at that point. Solution: look at the sign of $dx$, $dy$:

• $v_0, v_1$ is potentially entering the left edge if $dx = (x_1 - x_0) > 0$
• $v_0, v_1$ is potentially entering the right edge if $dx = (x_1 - x_0) < 0$ or $-dx > 0$

• $v_0, v_1$ is potentially entering the bottom edge if $dy = (y_1 - y_0) > 0$
• $v_0, v_1$ is potentially entering the top edge if $dy = (y_1 - y_0) < 0$ or $-dy > 0$
Liang-Barsky Algorithm

• Finding intersection type:
  – Entering left edge if $dx > 0$.
  – Entering right edge if $-dx > 0$.
  – Entering bottom edge if $dy > 0$.
  – Entering top edge if $-dy > 0$.

• Finding $t$:
  – For left edge: $t = \frac{x_{\text{left}} - x_0}{x_1 - x_0} = \frac{x_{\text{left}} - x_0}{dx}$
  – For right edge: $t = \frac{x_{\text{right}} - x_0}{x_1 - x_0} = \frac{x_{\text{right}} - x_0}{dx}$
    $= \frac{x_0 - x_{\text{right}}}{-dx}$
  – For bottom edge: $t = \frac{y_{\text{bottom}} - y_0}{y_1 - y_0} = \frac{y_{\text{left}} - y_0}{dy}$
  – For top edge: $t = \frac{y_{\text{top}} - y_0}{y_1 - y_0} = \frac{y_{\text{top}} - y_0}{dy}$
    $= \frac{y_0 - y_{\text{top}}}{-dy}$
Liang-Barsky Algorithm

- For lines parallel to edges:

```python
if d_x == 0 and x_min - x_0 > 0:  # left
    reject;
else if d_x == 0 and x_0 - x_max > 0:  # right
    reject;
else if d_y == 0 and y_min - y_0 > 0:  # bottom
    reject;
else if d_y == 0 and y_0 - y_max > 0:  # top
    reject;
```

\[ v_0 = (x_0, y_0) \]
\[ v_1 = (x_1, y_1) \]
Liang-Barsky Algorithm

• Putting it all together:

```c
bool visible(den, num, t_E, t_L):
  if (den > 0): // potentially entering
    t = num / den;
    if (t > t_L):
      return false;
    if (t > t_E)
      t_E = t;
  else if (den < 0): // potentially leaving
    t = num / den;
    if (t < t_E)
      t_E = t;
    if (t < t_L)
      t_L = t;
  else if num > 0: // line parallel to edge
    return false;
  return true;
```

t_E = 0; t_L = 1;
visible = false;
if visible(d_x, x_min - x_0, t_E, t_L): // left
  if visible (-d_x, x_0 - x_max, t_E, t_L): // right
    if visible (d_y, y_min - y_0, t_E, t_L): // bottom
      if visible (-d_y, y_0 - y_max, t_E, t_L): // top
        visible = true;
        if (t_L < 1):
          x_1 = x_0 + d_x * t_L;
          y_1 = y_0 + d_y * t_L;
        if (t_E > 0):
          x_0 = x_0 + d_x * t_E;
          y_0 = y_0 + d_y * t_E;
```
Liang-Barsky Algorithm

- 3D extension is straightforward:

\[ t_E = 0; \ t_L = 1; \]

visible = false;

**if** visible(d_x, x_min - x_0, t_E, t_L): // left

**if** visible (-d_x, x_0 - x_max, t_E, t_L): // right

**if** visible (d_y, y_min - y_0, t_E, t_L): // bottom

**if** visible (-d_y, y_0 - y_max, t_E, t_L): // top

**if** visible (d_z, z_min - z_0, t_E, t_L): // front

**if** visible (-d_z, z_0 - z_max, t_E, t_L): // back

visible = true;

**if** (t_L < 1):

\[ x_1 = x_0 + d_xt_L; \ y_1 = y_0 + d_yt_L; \ z_1 = z_0 + d_zt_L; \]

**if** (t_E > 0):

\[ x_0 = x_0 + d_xt_E; \ y_0 = y_0 + d_yt_E; \ z_0 = z_0 + d_yt_E; \]

This part is used for efficient ray-bounding volume intersections in acceleration structures we learned earlier!
Example

Left Edge

PE with small positive $t$ ($t_E = t$)

Right Edge

PL with large positive $t$ ($t_L = t$)
Example

**Bottom Edge**

PE with negative t ($t_E$ not updated)

**Top Edge**

PL with $t > 1$ ($t_L$ not updated)
Example

Largest $t_E$

Smallest $t_L$

Result
Polygon Clipping – Sutherland Hodgeman Algorithm

- Difficult problem as we need to deal with many cases:
Sutherland Hodgeman Algorithm

• Divide and conquer approach makes it manageable:
  – Solve a series of simple and identical problems
  – When combined, the overall problem is solved

• Here, the simple problem is to clip a polygon against a single clip edge:

Clip against right edge

\[ \text{Polygon} \rightarrow \text{Clipped Polygon} \]
Sutherland Hodgeman Algorithm

Clip against bottom edge

Clip against left edge
Sutherland Hodgeman Algorithm

Clip against top edge
Sutherland Hodgeman Algorithm

• This is accomplished by visiting the input vertices from \( v_0 \) to \( v_N \) and then back to \( v_0 \) for each clip boundary
• At every step we add 0, 1, or 2 vertices to the output:

Inside    Outside

Add \( v_{i+1} \) to output

Inside    Outside

Add \( v'_{i+1} \) to output

Inside    Outside

Add nothing to output

Inside    Outside

Add \( v'_{i+1} \) and \( v_{i+1} \) to output
Culling

• Complex scenes contain many objects
• Objects closer to the camera occlude objects further away
• Rendering time can be saved if these invisible objects are **culled** (i.e. eliminated, discarded, thrown away)
• Three common culling strategies are:
  – View volume (frustum) culling
  – Backface culling
  – Occlusion culling (advanced concept that we may learn later)
View Volume (Frustum) Culling

- The removal of geometry outside the viewing volume
- **No OpenGL support**: it is the programmer’s responsibility to cull what is outside.

From lighthouse3d.com
View Volume (Frustum) Culling

• First determine the equations of the planes that make up the boundary of the view volume (6 planes):

  Plane equation: \((p - a) \cdot n = 0\)

• Here, \(a\) is a point on the plane and \(n\) is the normal (pointing outside from the face)

• Plug the vertices of each primitive for \(p\). If we get:

  \((p - a) \cdot n > 0\)

  for any plane, the vertex is outside

• If all vertices are outside, then the primitive is outside and can be culled

• Using a bounding box or bounding sphere for complex models is a better solution.
Backface Culling

- For *closed* polygon models, back facing polygons are guaranteed to be occluded by front facing polygons (so they don’t need to be rendered)
- OpenGL supports backface culling: `glCullFace(GL_BACK)` and `glEnable(GL_CULL_FACE)`
Backface Culling

- Polygons whose normals face away from the eye are called back facing polygons.

Front facing triangle: $\mathbf{n} \cdot \mathbf{v} < 0$

Back facing triangle: $\mathbf{n} \cdot \mathbf{v} > 0$
Backface Culling

• Note the \( \mathbf{v} \) is the vector from the eye to any point on the polygon (you can take the polygon center). You cannot use the view vector!

From http://omega.di.unipi.it
Occlusion Culling

• The removal of geometry that is within the view volume but is occluded by other geometry closer to the camera:

• OpenGL supports occlusion queries to assist the user in occlusion culling
• By a fast rendering pass, it counts how many pixels of the tested object will be rendered
• This is commonly used in games (out of scope for this class)
Occlusion Culling in OpenGL

1. Create a query.
2. Disable rendering to screen (set the color mask of all channels to False).
3. Disable writing to depth buffer (just test against, but don't update, the depth buffer).
4. Issue query begin (which resets the counter of visible pixels).
5. "Render" the object's bounding box (it'll only do depth testing; pixels that pass depth testing will not be rendered on-screen because rendering and depth writing were disabled).
6. End query (stop counting visible pixels).
7. Enable rendering to screen.
8. Enable depth writing (if required).
9. Get query result (the number of "visible" pixels).
10. If the number of visible pixels is greater than 0 (or some threshold),
    - Render the complete object.

For details see: http://http.developer.nvidia.com/GPUGems/gpugems_ch29.html