ANT COLONY OPTIMIZATION

Swarm Inspired Methods

- Artificial life
- Social biological systems, group behaviour
- Swarm Intelligence
- Two approaches applying swarm intelligence to optimization problems
  - Ant Colony Optimization
  - Swarm Particle Optimization

Ant Colony Optimization

- Introduced first in 1991 by M. Dorigo, V. Maniezzo, and A. Colorni.
- Inspired from Ant searching for food and finding the shortest path back home.
- Randomly exploring the search space while using the elements from the previous solutions.

Combinatorial Optimization Problems

- $C = c_{1,} \dots, c_n$ basic components
  S a solution: subset of components
  $F \subseteq \wp(C)$ subset of feasible solutions
  $z : \wp(C) \rightarrow \mathbb{R}$ cost function
- find S^* such that
  $S^* \in F$, and $z(S^*) \leqslant z(S) \ \forall S \in F$

Basic Components of AOC

- A set of concurrent computation agents (Ants)
- Each ant moves based on a stochastic local decision based on:
  - trails (globally affected)
  - attractiveness (locally affected)
- Global mechanisms
  - trail evaporation
  - daemon actions

- Ants iteratively constructs the solution by local choices from state *i* to state *j*
- At each step *σ*, ant *k* computes a set of feasible expansions $A_k^{\sigma}(i)$ from its state.
- Probability of moving from state *i* to state *j* p_{ij}^{k} depends on:
  - Attractiveness η_{ij} of the move
  - trail level τ_{ij} of the move

Ant System

- The first idea applied
- Each move is based on a local probability value:

$$p_{i\,j}^{k} = \begin{cases} \dfrac{\tau_{i\,j}^{\alpha} + \eta_{i\,j}^{\beta}}{\displaystyle\sum_{(i\;l) \notin tabu_k} \left(\tau_{i\,l}^{\alpha} + \eta_{i\,l}^{\beta}\right)} & \text{if } (i\;\; j) \notin tabu_k \\ 0 & \text{otherwise} \end{cases}$$

- Trail levels are updated based on the tour length of previous solution and evaporation:

$$\tau_{i\ j}(t)=\rho\tau_{i\ j}(t-1)+\Delta\tau_{i\ j}$$

$$\Delta\tau_{i\ j}=\sum_{k=1}^{m}\Delta\tau_{i\ j}^{k}$$

$$\Delta\tau_{i\ j}^{k}=\begin{cases}\dfrac{Q}{L_k} & \text{if ant } k \text{ uses arc } (i\ \ j) \text{ in its tour}\\ 0 & \text{otherwise}\end{cases}$$

Any System Algorithm

1. Initialization
   Initialize $\tau_{i\,j}$ and $\eta_{\,i\,j}$ values

2. Construction
   foreach ant *k* (in state *i*) do:
   repeat
   choose the state j to move to (with prob.)
   Append the chosen move to tabu_k
   Until ant *k* has completed its solution

3. Trail update
   for each ant move (i j) do:
   compute $\Delta\tau(i\ j)$
   update trail matrix

4. Termination
   if not end of test, go to step 2

- Ant system could not produce competitive results:
  - All solutions, good or bad, contribute on the trails.
  - Pressure of transition probability is diverse (uniform probability or single edge takeover)
  - No local improvement
- Extensions:
  - Elitist Ant System (the elite updates trails at each cycle)
  - Ant Colony System
  - Max Min Ant System (max and min pheromone)

Ant Colony System

- 3 basic changes from Ant System:
  - Pheromone, trail update (best updates)
  - State transition rule (choice of exploration vs. exploitation)
  - Hybridization
- Competitive results (ie. known best values for TSP) found.

ACS: Pheromone

- Ant System: everyone updates pheromone trails globally.
- Ant Colony System: the current best since the beginning updates trails globally at each cycle.
- Search focuses around the current best.
- Local update and evaporation: only if an ant traverses that edge.

$$\tau_{i\ j}(t)=\rho\tau_{i\ j}(t-1)+(1-\rho)\tau_0$$

$$\tau_0=\frac{1}{nL_{nn}}\ ,\quad L_{nn}\text{: base (greedy) solution}$$

- Unvisited edges stays at τ_0 , visited gets smaller.
- Max-Min ant system uses similar upper-lower bounds.

ACS: Transition Rule

- Ant System: probability proportional to trail and attractiveness.
- Ant Colony System: *pseudo-random-proportional*.
- A choice is made among the best transition and probabilistically among others.

$$s=\begin{cases} max_{(i\ \ s)\notin tabu_k}\left\{\tau_{i\ \ j}^{\alpha}+\eta_{i\ \ j}^{\beta}\right\} & \text{if } q\leqslant q_0 \quad \textit{(exploit)} \\ \text{proportional to weighted values} & \text{otherwise} \quad \textit{(explore)} \end{cases}$$

q: random value such that $0\leqslant q\leqslant 1$

ACS: Hybridization

- Performing repairs on found individuals
- 2-opt, 3-opt, Lin-Kernighan
- Keep a list of preferred cities at each city (tabu search, etc).